The Measurement of Loan Termination Probabilities and Reverse Mortgage Insurer's Risks for the Korean Reverse Mortgage [*]

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Abstract: Korea has experienced the rapid aging society like most developed countries. Reverse mortgage actually is an important policy alternative to resolve the income shortage of elderly people through the liquidation of housing assets and the Korean government recently introduced the reverse mortgage system. However, it requires a long-term contract between lenders and borrowers. Therefore, the loan termination probability is one of key factors for implementing reverse mortgage programs successfully. The goal of this paper is to set up an appropriate mortality rate in order to figure out the loan termination rates and the reverse mortgage insurer's risk. The basic model of revere mortgage uses the 1.3 times of female's mortality rates as the loan termination probabilities and 100 years old as its age limitation like the U.S. HECM program because no termination experience is available in case of the initial premium setting. In order to figure out an appropriate loan termination probability and estimate the insurer's risk, this paper generates two different loan termination probabilities to reflect the real circumstance and also assumes 110-year life spans for confirming reverse mortgage insurer's risks. It notes that two different loan termination probabilities and the extension of maximum life span do not effect on the changes of the maximum levels of monthly payments considerably. So, we can conclude that the assumption of 1.3 times mortality rate can be applied into the Korean reverse mortgage market and can also be applied to other countries' reverse mortgage programs for setting the initial actuarial program.

1. Introduction

Korea has experienced the rapid aging society like most developed countries. Reverse mortgage actually is an important policy alternative to resolve the income shortage of elderly people through the liquidation of housing assets and the Korean government introduced the reverse mortgage system for the elderly at July 12, 2007. However, it requires a long-term contract between lenders and borrowers and the loan termination probability is one of the most important key factors for implementing reverse mortgage programs successfully. It eventually provides the widest array of cash-advance choices like tenure, term, credit lines, and their combinations. If borrowers select a tenure plan, they will receive a pre-determined amount of funds every month until they die, sell their home, or permanently move out. Therefore, termination probability is one of the most important factors in setting the actuarial model of reverse mortgages because it is directly related with the period of a reverse mortgage loan. U.S. HECM (Home Equity Conversion Mortgage) program, which is the first program which is guaranteed by the government, assumes that the borrower mortality rate is equal to 1.3 times of the age-specific female's mortality rate.
Many scholars (Booth, Hyndman, Tickle, and Jong, 2006; Denton, Feaver, and Spencer, 2005; Lazar, 2004; Rotger and Estany, 2002; Tuljapurkar, Li, and Boe, 2000) used the Lee-Carter methods for forecasting mortality rates and life expectancies since Lee-Carter introduced a new forecasting method (1992). Sung and Kim (2006) and Kang, Lee, and Sung (2006) in Korea also forecast mortality rates using Lee-Carter methods. However, Chow, Szymanoski, and DiVenti (2000), Rodda, Youn, Ly, Rodger, and Thompson (2003), and Szymanoski, Enriquez, and DiVenti (2006) argue

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that the 1.3 times mortality assumption underestimates termination probabilities at the younger ages, and overestimates them at the older ages. To alleviate these problems, Ma and Deng (2006) used male's mortality rate as female's termination rate in their analysis.
Three risk factors of reverse mortgage are loan termination probabilities, interest rate changes, and uncertainty of future housing price. However, the Korean government does not have the empirical data on the risk factors of reverse mortgage. Eventually the Korean reverse mortgage adopts the 1.3 times mortality assumption of the HECM in setting the initial program. And then the goals of this study are to set an appropriate loan termination probability and to figure out the insurer's risk in the Korean reverse mortgage market where the reverse mortgage is introduced firstly. In order to do that, it is necessary to evaluate the possible effects resulting from the assumption of loan termination in advance. Moreover, reverse mortgage backed securities also should be issued in order to guarantee the continuous reverse mortgage payment to borrowers.
KNSO (Korea National Statistical Office) released 2003 mortality table in the end of 2005 and did 2004 and 2005 mortality tables in December 2006. The 2007 mortality table is expected to be announced in the end of 2008. Therefore, in order to decide the termination probabilities of 2007 reverse mortgage actuarial model, this study should forecast 2007 mortality rates using these released data.
The Korean government-insured reverse mortgage assumes that the mortality rates of aged 100 and above became 1.0 (the oldest possible survival age is 100). This study will forecast 2007 mortality rates using Lee-Carter model and decide the 2007 loan termination probabilities according to 1.3 times mortality assumption. And then this paper generates two different loan termination probabilities reflecting the results of empirical studies in the U.S. HECM program. Each model also includes the assumption which the maximum life span is extended to 110 years old for confirming the magnitude of reverse mortgage insurer's risks resulting from borrowers' longevity. The second section notes a basic actuarial model of reverse mortgage, section 3 figures out the loan termination probabilities, section 4 forecasts the mortality rates and analyzes the insurer's risks resulting from loan termination probability assumptions.

2. Basic Actuarial Model of Reverse Mortgage System

The Korean government introduced a reverse mortgage system which borrowers are required to pay 2% of housing values as an upfront mortgage insurance premium (UP_0) and a monthly mortgage insurance premium (mip_t) according to the annual rate of 0.5% of the outstanding loan balances in July 12, 2007. And the maximum amounts of constant monthly payments are evaluated using trial and error method under the condition that the present values of expected claim losses (PVEL) are equal to that of insurance premiums (PVMIP) as Eq(1).

$$PVMIP = UP_0 + \sum_{t=1}^{T(a)} \left[\frac{mip_t \cdot p_{a,t}}{(1+i)^t} \right]$$
$$= \sum_{t=1}^{T(a)} \left\{ \frac{\max\left[(OLB_t - H_t) \cdot q_{a+t}, 0 \right] \cdot p_{a,t}}{(1+i)^t} \right\} = PVEL \qquad (1)$$

Where

$PVMIP$	≡	present value of total mortgage insurance premiums at time t=0
$PVEL$	≡	present value of total claim losses at time t=0
UP_0	≡	upfront mortgage insurance premium at time t=0
$T(a)$	≡	the number of months that borrowers with age a will continue to receive the annuity payments until they reach to age 100
mip_t	≡	monthly mortgage insurance premiums at time t $mip_t = (OLB_{t-1} + payment) \cdot m$ $payment$ = constant monthly payment; m =monthly premium rate
OLB_t	≡	the outstanding loan balance at time t $OLB_t = [(OLB_{t-1} + payment + mip_t)](1+i)$
$p_{a,t}$	≡	the probability that a borrower of age a will survive at age $a+t$
i	≡	expected interest rate
H_t	≡	expected housing values at time t $H_t = H_0 \cdot (1+g)^t$; g = mean value of housing price growth rate

q_{a+t} ≡ the probability that a loan will be terminated at age $a+t$

According to Ma, Kim, and Lew (2007), if this study assumes that the values of $UP_0 = H_0 \times 2\%$, $m = 0.5\%/12$, $i = 7.5\%$, $g = 3.5\%$, and q_{a+t} = 1.3 times of female's mortality rate, it can measure the present values of total mortgage insurance premiums (PVMIP) and those of total claim losses (PVEL) at time t=0 and determine the maximum level of constant monthly payments (annuities) like Table 1.

Table 1. Determining Maximum Level of Monthly Payment (unit: Korean Won ($ 1/930))

PVEL	PVMIP	Net Liability	Constant Monthly Payment
11,605,962	11,606,481	-519	1,067,950

Note: 1. Net Liability=PVEL-PVMIP
2. Borrower's age: 70
3. Initial housing value (H_0): 300 millions won
4. 930 won in Korea is about 1.0 dollar in the U.S.

This study can confirm that the maximum level of constant monthly payment is 1,067,950 won.

3. Loan Termination Probabilities of Reverse Mortgage

This study can measure loan termination probabilities and loan survival probabilities like Eq (2).

$$s_{a,t} = \prod_{j=1}^{t} \left(1 - d_{a+j}\right) \quad (2)$$

where $s_{a,t}$ = loan survival probability in time *t* of borrower's age a

$\prod$ = product operator

d_{a+t} = loan termination probability in borrower's age $a+t$

Loan survival probabilities ($s_{a,t}$) in Eq (2) can be measured as follows (Szymanoski, Enriquez, and DiVenti, 2006).

$$s_{a,0} = 1.0,$$
$$s_{a,1} = s_{a,0} \cdot \left(1 - d_{a+1}\right),$$
$$s_{a,2} = s_{a,1} \cdot \left(1 - d_{a+2}\right) = s_{a,0} \cdot \left(1 - d_{a+1}\right) \cdot \left(1 - d_{a+2}\right) = \prod_{j=1}^{2} \left(1 - d_{a+j}\right),$$

and it can be generalized as follow;

$$s_{a,t} = s_{a,t-1} \cdot \left(1 - d_{a+t}\right) = \prod_{j=1}^{t} \left(1 - d_{a+j}\right).$$

Loan survival probabilities are used for evaluating both the present values of expected claim losses and those of expected insurance premiums in the actuarial model of reverse mortgage. Yet, loan termination probabilities are used for evaluating the present values of expected claim losses. The HECM measured the expected insurance premium and claim losses using loan survival probabilities $s_{a,t}$ which are measured by the assumption which the loan termination probabilities d_{a+t} at age *a+t* are the 1.3 times of female mortality rates (30% of female mortality are assumed as the advanced repayment rates by the other reasons except the death of borrowers). Based upon the condition which the present values of expected insurance premium are equal to those of expected claim losses, the HECM decide the maximum level of monthly payment for borrowers. In order to apply these annual loan survival probabilities ($S_{i,j}$) data to the basic actuarial model of reverse mortgage, the HECM program converted these data into the monthly loan survival probabilities ($p_{a,t}$) by using the geometrical interpolated methods (Szymanoski, 1990) as follows;

$$p_{a,t} = \left[S_{i,j} \cdot \left(\frac{S_{i,j+1}}{S_{i,j}} \right)^{\frac{r}{12}} \right]^{1+m} \tag{3}$$

where	i	=	initial age in years = {62, 63, …, 99}
	j	=	attained age in full years = { i , i +1, …, 100}
	a	=	initial age in months = 12 i
	t	=	attained age minus initial age in months = 12(j - 1)+ r
	r	=	months between attained ages j and $j+1$ = {0,1, …, 11}
	m	=	move-out rate expressed as a decimal = 0.3

If this paper simply follows the method of HECM, the monthly loan survival rates (termination rates) might be a little higher (lower) than those of their actual values because the annual mortality rates of KNSO represent the median value of each age interval. Due to this reason, this study used the smoothed Hodrick-Prescott (HP) trends for interpolating the value of mortality rates, following Ma and Deng (2006). The HP filter is a two-sided linear filter that computes the smoothed series s of y by minimizing the variance of y around s, subject to a penalty that constrains the second difference of s. That is, the Hodrick-Prescott filter chooses s to minimize $\sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} [(s_{t+1} - s_t) - (s_t - s_{t-1})]^2$. The penalty parameter λ controls the smoothness of the series s. The larger the λ , the smoother the s. As $\lambda \to \infty$, s approaches a linear trend (Hodrick and Prescott, 1997).

4. Methodology, Data and Forecasting Mortality Rates

4-1. *Methodology*

The model of Lee-Carter for forecasting mortality rates combines a demographic model of mortality with time series methods of forecasting like Eq(4).

$$ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \qquad (x = 1,2,\cdots,n; t = 1,2,\cdots,T) \tag{4}$$

$$k_t \sim I(d)$$

$$\text{but, } \sum_{x=1}^{n} b_x = 1, \qquad \sum_{t=1}^{T} k_t = 0$$

where	$m_{x,t}$	=	mortality rate at age group x in year t
	a_x	=	average pattern of mortality by age group across years
	b_x	=	the relative change slope at each age group by the change of mortality rates
	k_t	=	mortality index at time t
	$\varepsilon_{x,t}$	=	residual at age group x and time t

In equation (4), the estimators of a_x are given by the sample mean of the natural logs of the mortality rates like Eq (5).

$$a_x = T^{-1} \cdot \sum_{t=1}^{T} ln(m_{x,t}); \qquad x = 1,2,\cdots,n \tag{5}$$

The singular value decomposition method is used to estimate the parameters b_x and k_t. The estimation of k_t is obtained by summing the deviations of the logs of the mortality rates across all the age groups like Eq (6).

$$k_t = \sum_{x=1}^{n} \left[ln(m_{x,t}) - a_x \right] \tag{6}$$

And b_x are estimated regressing the deviations of the logs of the mortality rates with the initial estimation of k_t like Eq (7).

$$b_x \Big| \; LS\left[ln(m_{x,t}) - a_x \right] \; on \; k_t . \tag{7}$$

A random walk model with drift (that is ARIMA (0,1,0) model) in general is used for forecasting mortality index (k_t). However, this study found that ARIMA(0,1,0) model is inadequate for forecasting k_t series in Korea because the first differenced time series of k_t was non-stationary. And this study used Holt-Winters' no seasonal exponential smoothing method for forecasting mortality index. Smoothed time series $\hat{k}_t$ in HW no seasonal exponential smoothing method is like Eq (8).

$$\hat{k}_{t+y} = a + y \cdot b \tag{8}$$

where a = intercept, b =slope

We can get the values of a_t and b_t at time t as follows.

$$\begin{aligned} a_t &= \alpha k_t + (1-\alpha)(a_{t-1} + b_{t-1}), \quad 0 < \alpha < 1 \\ b_t &= \beta(a_t - a_{t-1}) + (1-\beta)b_{t-1}, \quad 0 < \beta < 1 \end{aligned} \tag{9}$$

Now, we can forecast the value of mortality index (k_t) after period y from time T as follows.

$$\hat{k}_{T+y} = a_T + y \cdot b_T \tag{10}$$

4-2. *Data*

It is necessary to use the time series of mortality rates of year based age group in the life tables for males and females in order to forecast future mortality rates using the Lee-Carter model. However, this study forecasts the future mortality rates and the loan termination probabilities of reverse mortgage, using the time series data of the simplified 5-year age interval life table from 1971 to 2005 (35 years) which were released by Korea National Statistical Office (KNSO) because there is no sufficient time series data on the year based mortality rates. 5-year age interval mortality rates can be expressed like Eq (11).

$$_nQ_x = \frac{_nD_x}{L_x} \tag{11}$$

where $\quad _nQ_x \quad = \quad$ Mortality rates by 5-year age intervals

$_{n}D_{x}$ = The number of deaths by 5-year age intervals

$_{n}D_{x} = \sum D_{x}$ (D_{x} = the number of deaths of each age)

L_{x} = The number of survivors of each age

There are 17 age groups like Table 2 but this study will forecast the age groups from age group 18^{th} to 22^{th}.

Table 2. Definition of 5 year interval age groups on the male and female mortalities

Age group: x	symbol	Age range	Age group: x	symbol	Age range
1	Q00	Age 0	12	Q50	Age 50-54
2	Q01	Age 01-04	13	Q55	Age 55-59
3	Q05	Age 05-09	14	Q60	Age 60-64
4	Q10	Age 10-14	15	Q65	Age 65-69
5	Q15	Age 15-19	16	Q70	Age 70-74
6	Q20	Age 20-24	17	Q75	Age 75-79
7	Q25	Age 25-29	18	Q80	Age 80-84
8	Q30	Age 30-34	19	Q85	Age 85-89
9	Q35	Age 35-39	20	Q90	Age 90-94
10	Q40	Age 40-44	21	Q95	Age 95-99
11	Q45	Age 45-49	22	Q100	Age 100+

Figure 1 shows the trends of mortality rates for males and females from 1971 to 2005. It notes that the mortality rates of the male and female from 13^{th} to 17^{th} age group go downward by passage of time.

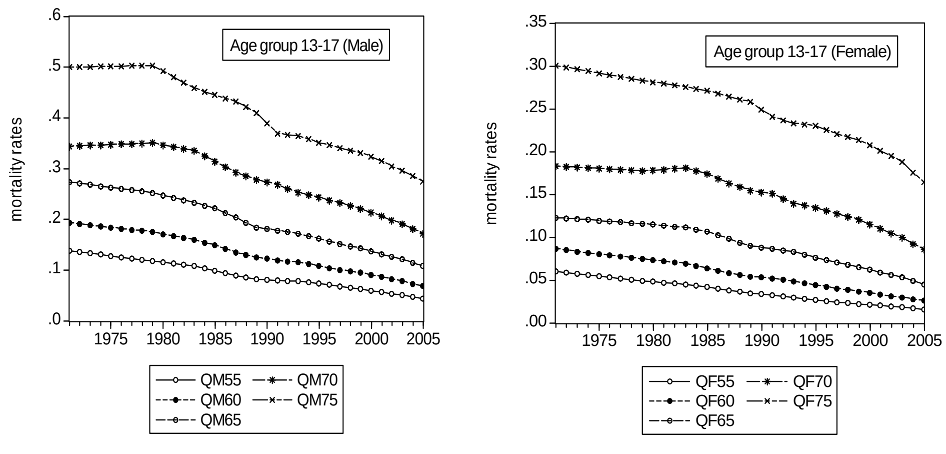

Figure 1. Trends of mortality rates for males and females

4-3. *Forecasting mortality rates*

Table 3 shows the estimated values of a_x and b_x for females by age group x, based upon Lee-Carter model.

Table 3. Estimated values of a_x and b_x for females

age group (x)	a_x	b_x	age group (x)	a_x	b_x
1	-4.25151	0.082721	12	-3.74124	0.047554
2	-5.40542	0.087626	13	-3.36685	0.043562
3	-5.81141	0.092768	14	-2.92151	0.038931
4	-6.11251	0.086615	15	-2.42597	0.031782
5	-5.65849	0.076238	16	-1.90949	0.022911
6	-5.40657	0.076189	17	-1.39738	0.017218
7	-5.28883	0.072537	18	-0.96424	0.017218
8	-5.1114	0.064031	19	-0.51868	0.017218
9	-4.8437	0.056021	20	-0.20371	0.017218
10	-4.49572	0.053469	21	-0.01013	0.017218
11	-4.11521	0.049828			

Table 3 notes the sum of estimated parameter b_x from age group 1th to 17th satisfying the condition of $\sum_{x=1}^{17} b_x = 1$. The values of a_x and b_x after age group 17th are used for forecasting the future mortality rates. Following Rotger and Estany (2002), this study forecasted the mortality rates after the age group 17th, using a fixed value of b_x like Table 3. Meantime, this study generated the values of a_x considering the real mortality rates in the 2005 life table of KNSO which has 5-year age intervals. This life table shows the real mortality rates from age group 18th to age group 22th, respectively. Table 4 shows mortality indexes estimated using data from 1971 to 2005 and the projected 2006 and 2007 mortality indexes.

Table 4. Mortality index (k_t) for females

year	k_t	year	k_t	year	k_t	year	k_t
1971	13.68873	1981	7.1460	1991	-2.80275	2001	-12.0102
1972	13.07473	1982	6.326938	1992	-3.83682	2002	-12.9036
1973	12.46189	1983	5.509792	1993	-4.86078	2003	-13.8043
1974	11.84716	1984	4.382132	1994	-5.5087	2004	-14.3322
1975	11.23425	1985	3.25494	1995	-6.17383	2005	-14.9288
1976	10.61992	1986	2.211432	1996	-7.05727	2006	-15.5225
1977	10.00603	1987	1.163098	1997	-7.95478	2007	-16.1161
1978	9.393576	1988	0.095732	1998	-9.01363		
1979	8.781195	1989	-0.96922	1999	-10.0677		
1980	7.962269	1990	-1.88919	2000	-11.046		

It also notes that the sum of estimated parameter k_x from 1971 to 2005 satisfies the condition of $\sum_{t=1}^{T} b_x = 0$. This study forecasts 2006 and 2007 mortality indexes using HW exponential smoothing method and Table 5 shows the results of estimating parameters.

Table 5. Results of parameter estimations of mortality rates

	α	β	a_T	b_T	SSR	RMSE
k_t for females	1.0	0.93	-14.9288	-0.5936	0.5853	0.1293

Note: SSR = sum of squared residuals; RMSE = root mean squared error

Figure 2 shows the estimated and forecasted values of a_x and b_x for females using data from 1971 to 2005.

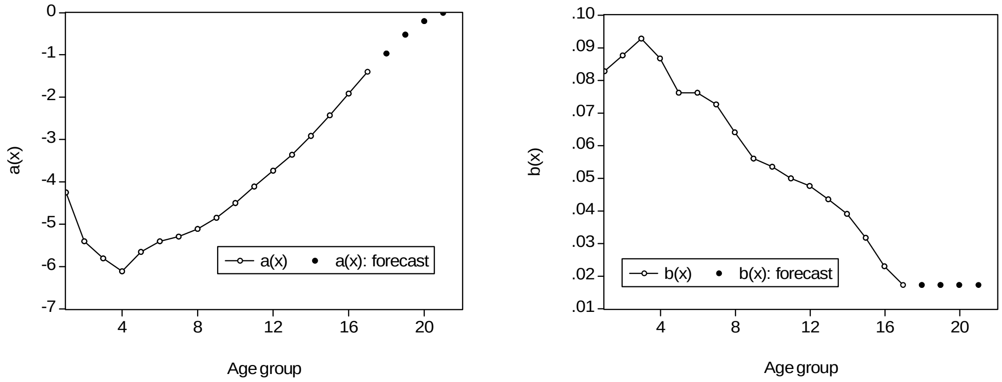

Figure 2. estimated values of a_x and b_x for females

Figure 3 shows the estimated and forecasted values of k_x for females in the same period.

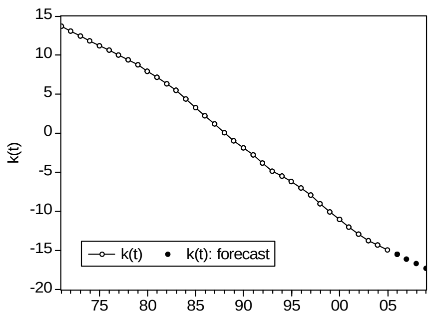

Figure 3. Estimated values of k_x for females

Now, this study can forecast the values of 5 year interval mortality rates for females in 2007, using the estimated and forecasted values of a_x, b_x and k_x. Figure 4 shows 2005 and 2007 mortality rates for females (mort05 and mort07) and 1.3 times of 2007 females' mortality rates (mort07*130%) and 1.25 times of 2005 females' mortality rates (mort05*125%).

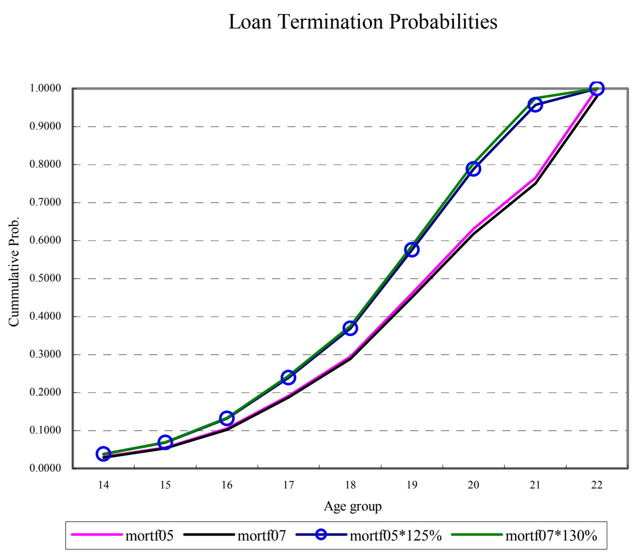

Figure 4. Comparison of mortality rates from age group 14 to 22

Figure 4 shows that mortality rates in 2007 are slightly smaller than them in 2005 as expected. It also notes that the values of 1.25 times of 2005 females' mortality rates are located just below those of 1.3 times of 2007 females' mortality rates. So, we can see that there are no significant differences between these two mortality rates. Consequently the values of 1.25 times of 2005 females' mortality rates (mort05*125%) are appropriate as the proxy variable of the loan termination probabilities in the actuarial model setting of the Korean reverse mortgage market where the reverse mortgage is just introduced and there are no periodical data on the loan termination probabilities. From now on, we can use the values of 1.25 times of 2005 females' mortality rates as loan termination probabilities. In other words, this paper assumes that the values of 1.25 times of 2005 females' mortality rates have the same values of 1.3 times of 2007 females' mortality rates (mort07*130%).

5. Analysis of Insurer's Risks

This study analyzed reverse mortgage insurer's risks resulting from loan termination probabilities in the case of borrowers' age 70. Based upon the above analysis, it used the values of 1.25/1.30 times of females' mortality rates in 2005 as a proxy variable of each year mortality rate for females in 2007 because there was no data on the mortality rates in July 12, 2007 when the reverse mortgage was introduced. In order to convert this 2007 female mortality rates into monthly mortality rates, this study used the smoothed HP filter. As the previous description, the HECM program assumed that the probability of prepayment of reverse mortgage except the death of borrowers is 0.3 times of female's mortality rates. Therefore, the total loan termination probabilities of reverse mortgage loans became 1.3 times of female's mortality rates. However, the recent empirical studies noted that this loan termination assumption did not reflect the actual reverse mortgage market situations. Chow, Szimanoski, and DiVenti (2000), Rodda, Youn, Ly, Rodger, and Thompson (2003), and Szimanoski, Enriquez, and DiVenti (2006) argued that it under-predicted loan termination probability at younger ages, and over-predicted them at older ages in terms of the periodical aspect. In order to solve the above problems, a quadratic polynomial trend model is used for generating new loan termination probabilities. At first, it estimated 2007 female's mortality rates using quadratic polynomial trend model and then generated two different loan termination probabilities after modification of the parameters. More specifically it modified that the values of loan termination probabilities became greater than the values of 1.3 times mortality rate assumption at the younger ages and this study modified loan termination probabilities became lower than the values of 1.3 times mortality rate assumption at the older ages as follows. Table 6 shows the estimated parameters of 2007 female's mortality rates (q_x) and that of two different mortality rates (q_x_mod1 and q_x_mod2) for evaluating the risks of reverse mortgages.

Table 6. Estimated quadratic polynomial trend models

loan termination probabilities	β_0	β_1	β_2
q_x	0.011891	0.001274	0.000269
q_x_mod1	0.040000	0.000500	0.000265
q_x_mod2	0.070000	-0.002000	0.000315

Note: quadratic polynomial trend model: $y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon_t \quad (t = 0, 1, 2, \cdots, T)$

Figure 5 shows q_x, q_x*1.3, q_x_mod1, and q_x_mod2, respectively.

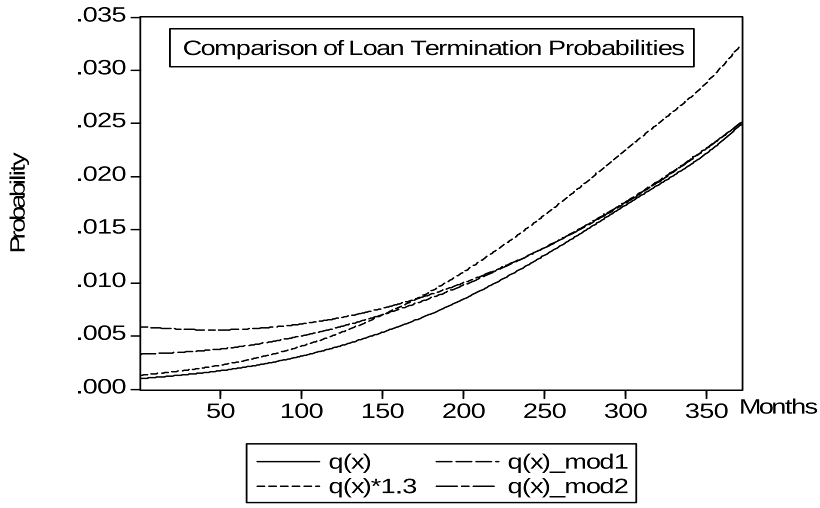

Figure 5. Comparison of 4 loan termination probabilities

Figure 5 shows that the values of new generated q_x_mod1 and q_x_mod2 present more realistic shapes in the loan termination probabilities reflecting the results of recent empirical studies as the above description. Based upon the values of q_x, q_x*1.3, q_x_mod1, and q_x_mod2, it measured the levels of monthly payments. And it also analyzes the effect of maximum life span which the oldest possible survival age is extended to 110 years old. Table 7 shows the values of monthly payments which satisfies the actuarial equivalence principle.

Table 7. Monthly payments which satisfies the actuarial equivalence principle

[Maximum life span: age 100]					
	PVEL	PVMIP	Net Liability	Monthly payment	Ratio to Basic model
q_x	12,264,818	12,265,713	-895	914,850	0.899
q_x*1.3	**11,819,186**	**11,819,473**	**-287**	**1,017,810**	**1.000**
q_x_mod1	11,150,677	11,151,652	-975	1,002,180	0.985
q_x_mod2	10,450,424	10,450,876	-451	1,056,640	1.038
[Maximum life span: age 110]					
	PVEL	PVMIP	Net Liability	Monthly payment	Ratio to Basic model
q_x	12,277,079	12,277,386	-307	906,340	0.890
q_x*1.3	11,819,379	11,820,005	-626	1,014,170	0.996
q_x_mod1	11,161,383	11,161,917	-533	995,060	0.978
q_x_mod2	10,460,573	10,461,086	-513	1,049,860	1.031

Note: 1. Borrower's age: 70
2. Initial housing value: 300,000,000 won (930 won in Korea is about 1.0 dollar in the U.S.)
3. Basic model: Maximum life span is age 100 and loan termination rate is q_x*1.3

Under the condition of the actuarial equivalence principle the basic model of Korean reverse mortgage (the age limitation is 100 and loan termination probability is (q_x*1.3), presents 1,017,810 won in the maximum monthly payment amount, and q_x_mod1 and q_x_mod2 also note 1,002,180 (98% of basic model) and 1,056,640 (104%) respectively. The modified models (q_x_mod1 and q_x_mod2; which have 110 of the age limitation), also show the

amounts of maximum monthly payment and their ratios to the basic model. This analysis confirms that there are no considerable differences in the values of monthly payments between the basic model (q_x*1.3) and the modified models (q_x_mod1, and q_x_mod2). It means that the extension of maximum life span does not change the maximum levels of monthly payments significantly.
In order to figure out the differences of the insurer's risks according to the assumptions of loan termination probabilities, this study analyzes the magnitude of present values of insurer's net insurance liabilities. Table 8 shows the net liabilities among the different models.

Table 8. Comparison of insurer's net insurance liabilities

[Maximum life span: age 100]				
	PVEL	PVMIP	**Net Liability**	Monthly payment
q_x	17,994,784	12,913,400	5,081,383	1,017,810
q_x*1.3	**11,819,186**	**11,819,473**	**-287**	**1,017,810**
q_x_mod1	11,777,269	11,225,587	551,682	1,017,810
q_x_mod2	9,184,082	10,300,221	-1,116,138	1,017,810
[Maximum life span: age 110]				
	PVEL	PVMIP	**Net Liability**	Monthly payment
q_x	18,456,075	12,986,275	5,469,800	1,017,810
q_x*1.3	11,985,117	11,839,254	145,863	1,017,810
q_x_mod1	12,071,014	11,270,496	800,518	1,017,810
q_x_mod2	9,417,449	10,335,669	-918,219	1,017,810

Table 8 shows the fact that the values of reverse mortgage insurer's net insurance liabilities would become both positive (+) or negative (-) according to the assumptions of loan termination probabilities, comparing these results with that of basic model. However, the differences of net insurance liabilities are not large considerably.

6. Conclusion

The reverse mortgage program becomes popular recently because the Korea has experienced the rapid aging society and the pension and other social security systems for the elderly are weak relatively. And the Korean government introduced the government-insured reverse mortgage system in July 12, 2007.
The tenure reverse mortgage loans will be terminated when the borrower dies leaves her home permanently or simply chooses to pay off the outstanding loan balances. The HECM program assumes that the probability of other prepayment except the death of borrower is 0.3 times of female's mortality rates. Therefore, the total loan termination probability of reverse mortgage loans assumes to be 1.3 times of female's mortality rates. This study aimed to figure out appropriate loan termination probabilities and evaluate the insurers' risk in the Korean reverse market where the reverse mortgage program is introduced firstly and there are no empirical data on the loan termination probabilities and insurers' risk. Using the model of government-insured reverse mortgage (HECM) of U.S.A., this study forecasted 2007 mortality rates for females using Lee-Carter model and then generated two different loan termination probabilities in order to resolve the problems of previous studies and to reflect the real reverse mortgage market. It also evaluates the net liabilities among different models. For generating new loan termination probabilities, this study estimated 2007 female's mortality rates using quadratic polynomial trend model and then generated two different loan termination probabilities after modification of the parameters. And then this study also conducted additional analysis for confirming the effect of maximum life span. This study used the quadratic polynomial trend model for forecasting the values of loan termination probabilities after age 100. It noted that there were no considerable differences in the values of monthly payments among the basic model (q_x*1.3), q_x_mod1, and q_x_mod2 and also figured out that the extension of maximum life span did not effect on the change of the maximum levels of monthly payments considerably.
To confirm insurer's risks due to the change of loan termination probabilities, this study measured the present values of insurer's net insurance liabilities. This study confirmed that there were no considerable differences in the values of insurer's net liabilities among the models. Consequently, although 1.3 times mortality rate assumption could not explain real world situation, this study could find that 1.3 times assumption would not violate the condition of actuarial

equivalence principle significantly. Moreover, q_x_mod1, and q_x_mod2 contributed to resolve these problems but the analysis results also did not change significantly. So, this study can conclude that the assumption of 1.3 times mortality rate can get external volatility in the country which has no loan termination probabilities and insurers' risk because the reverse mortgage program is introduced firstly. Furthermore, this study also can contribute to forecast the mortality rates, loan termination probabilities, and insurers' risks resulting from the assumptions of loan termination probabilities for setting the new reverse mortgage program. Its results can be applied to other countries' program where reverse mortgage is introduced to solve the income shortage of the elderly homeowners.

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